PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Storm Surges in the North Sea, 11 to 30 December 1954

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STORM SURGES IN THE NORTH SEA, 11 TO 30 DECEMBER 1954

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CONTENTS

The disturbances produced by the stormy conditions of December 1954 have been analyzed. The basic data used were tidal observations at ten British and fifteen continental stations in the North Sea and English Channel, together with records of the mean flow of water through the Straits of Dover as represented by electromagnetically induced currents in a telephone cable running from St Margaret's Bay to Sangatte.

The main purpose of the investigation has been to determine the relative magnitudes of the various factors contributing to the phenomena. The effect of westerly winds has been shown to depend upon whether the wind system is confined to the North Sea, or to the north-western approaches to the sea, or is a broad airstream covering both areas. Evidence has been put forward for the existence of a 'return' surge, or southward return of water previously expelled from the North Sea, on 15 December.

Co-disturbance charts have been constructed for the large surges of 20 to 25 December, and the water movements thus deduced exhibit marked geostrophic effects in all cases. An example of an external surge has been noted. Representing the sea by a rectangle, a correlation of 0·96 was found between the longitudinal water gradient and the geostrophic wind; this analysis led to a value of the wind stress coefficient, γ^2, of $2{\cdot}7\times10^{-3}$. The transverse gradient has been shown to be composed of a direct wind effect and a larger geostrophic effect. Estimates have been made of the mean level of the sea which, during surge peaks, was some 2·5 ft. above normal, and these have been represented satisfactorily by winds in and to the north of the sea.

The cable measurements throughout the whole period have been analyzed at intervals of 25 h, and, after Bowden (1956), provided estimates of γ^2 ($2{\cdot}1\times10^{-3}$) and the bottom friction coefficient k ($3{\cdot}5\times10^{-3}$). A similar analysis of the data at 3 h intervals gave values of $2{\cdot}3\times10^{-3}$ and $4{\cdot}5\times10^{-3}$, respectively.

The subject of the oscillatory development of storm surges has been reviewed with particular reference to the North Sea, and the conclusion reached that for this area the motion appears to be so heavily damped that positive surges may be represented, for practical purposes, by equilibrium conditions modified by a time lag. Negative surges, however, exhibit a strong tendency towards oscillations, as evidenced by the existence of return surges.

1. Introduction

In accordance with certain recommendations made by the Waverley Committee, which was set up in 1953 as a result of the East Coast flooding disaster in January of that year, the Tidal Institute has in recent years conducted intensive investigations into the nature of storm surges and means of forecasting them. The work has covered the North Sea, English Channel and Irish Sea areas, and frequent reports have been submitted to the Advisory Committee for Oceanographical and Meteorological Research, under the Ministry of Agriculture, Fisheries and Food, which is in charge of the research. The results have not as yet been published, and where they enter into the discussions to follow they will be referred to by [R]. The present paper is part of the series of investigations.

Whilst it is possible to obtain satisfactory correlations between residuals at any one port and suitable meteorological variables, and hence to formulate a method of predicting surges from meteorological forecasts, only a limited knowledge is acquired thereby of the physical processes involved for an area such as the North Sea. A regional investigation for a suitably chosen period is the best means so far devised for studying the water movements associated with surges, and for examining the relative importance of the many factors contributing to the phenomena. For the North Sea we may summarize these contributory factors as follows:

(*a*) Statical barometric pressure effect.

(*b*) Local wind effect due to shallow water and coastal topography.

(*c*) External surges.

(*d*) 'Return' surges.

(*e*) Wind drift southward into the sea, generated by northerly winds near the entrance to the sea and by westerly winds to the north or north-west of the British Isles, both resulting in changes in the mean level of the sea.

(*f*) A longitudinal water gradient generated by winds inside and to the north of the sea.

(*g*) A transverse water gradient generated by winds inside the sea, and also by geostrophic forces.

(*h*) Outflow or inflow through the Straits of Dover, principally affecting levels in the southern arm of the sea.

(*i*) Oscillatory motion.

(*j*) Interaction between tide and surge.

In this paper the theoretical law for (*a*) has been presumed valid, and the factor (*b*) has been minimized wherever possible when manipulating the data. External surges have received considerable attention in this country (Corkan 1948; Goldsbrough 1952; Crease 1956 *a*, *b*; and [R]) but do not figure prominently in the period discussed here. Consideration of the interaction between tide and surge will be restricted to indicating, in § 2, the areas in which this phenomenon becomes significant. The importance of the remaining factors (*d*) to (*i*) will be examined in this paper for the period 12 to 28 December. This was a very stormy period in the North Sea, particularly from 20 to 24 December when two large surges were experienced. These were especially dangerous in the German Bight, and have attracted the attention of Tomczak (1955) and Weenink (1956).

2. Reduction of tidal records and presentation of results

Tidal observations for ports around the North Sea were supplied either in the form of automatic gauge records or as hourly tabulations from such records. A list of stations and the authorities supplying the data is given in table 1. The problems involved in eliminating the astronomical tide to the required degree of accuracy when harmonic constants are not known, or where shallow-water complications arise, has recently been re-examined at the Tidal Institute, and the method given by Doodson (1929) has been improved upon. It is

Table 1. Observational stations and corrections to mean residuals

(a) place	(b) lat. (°)	(b) lat. (′)	(c) long. (°)	(c) long. (′)	(d) authority for observations	(e) ft.	(e) ft.	(f) ft.	(g) ft.
Aberdeen	57	09 N	2	05 W	Harbour Engineer, Aberdeen	0·2	—		−0·1
Leith	55	59 N	3	10 W	Leith Dock Commission	0·2	0·2	0·0	−0·1
Tynemouth	55	01 N	1	24 W	Tyne Improvement Commission	0·2	0·0	0·2	0·1
R. Tees entrance	54	38 N	1	09 W	Tees Conservancy Commission	0·3	0·3	−0·1	0·0
Grimsby	53	35 N	0	04 W	British Transport Commission	0·2	−0·2	0·5	0·5
Lowestoft	52	28 N	1	45 E	British Transport Commission	1·0	0·9	−0·8	−0·3
Harwich	51	57 N	1	17 E	Harwich Conservancy Board	0·5	0·4	0·5	0·2
Southend	51	31 N	0	45 E	Port of London Authority	0·6	0·6	−0·1	0·1
Dover	51	07 N	1	19 E	Dover Harbour Board	−0·3	−0·3	0·9	1·0
Newhaven	50	47 N	0	30 E	British Transport Commission	0·4		−0·7	0·3
Dover	—		—		—	−0·3			1·0
Ostend	—		—		—	0·4		−0·7	0·3
Dieppe	49	56 N	1	06 E	Service Centrale Hydrographique	−0·7			1·3
Ostend	51	14 N	2	55 E	Hydrographic Service	0·3	0·4	−1·0	0·3
Brouwershavn	51	44 N	3	54 E	Rijkswaterstaat	0·0	0·0	0·4	0·7
Ijmuiden	52	28 N	4	34 E	Rijkswaterstaat	0·1	0·3	−0·1	0·6
Terschelling	53	22 N	5	13 E	Rijkswaterstaat	−0·1	0·2	0·1	0·7
Borkum	53	35 N	6	39 E	Deutsches Hydrographisches Institut	0·8	0·4	−0·9	−0·2
Norderney	53	42 N	7	10 E	Deutsches Hydrographisches Institut	0·9	0·7	−0·3	−0·5
Büsum	54	08 N	8	51 E	Deutsches Hydrographisches Institut	1·1	1·2	−0·2	−0·7
List	55	01 N	8	27 E	Deutsches Hydrographisches Institut	1·1	1·1	0·1	−0·6
Esbjaerg	55	28 N	8	27 E	Det Danske Meteorologiske Institut	1·4	1·6	−0·3	−0·9
Thyborøn	56	42 N	8	14 E	Det Danske Meteorologiske Institut	1·5	1·5	0·1	−0·8
Tregde	58	00 N	7	34 E	Norges Geografiske Oppmåling	0·4	−0·1	1·1	0·3
Stavanger	58	58 N	5	44 E	Norges Geografiske Oppmåling	0·2	0·2	−0·3	0·0
Bergen	60	24 N	5	18 E	Norges Geografiske Oppmåling	−0·1	0·0	0·3	0·3
Målöy	61	56 N	5	07 E	Norges Sjøkartverk	—	0·0	0·0	0·3

See text for definitions of columns (e) to (g).

hoped that the method now developed which has been extensively employed by the Tidal Institute will shortly be published. For the purpose of this paper it will suffice to say that the residuals (observed minus 'predicted' tide) have been most carefully extracted and meet the requirements of the present investigation from the standpoint of accuracy.

Assuming the theoretical, statical relationship between changes in water level and atmospheric pressure, the residuals have been corrected for this local barometric effect.

In the absence of a reliable knowledge of the height of mean sea level relative to the datum of observations at many of the ports, and the lack of a relationship between the land-levelling systems of England and the continent, the data themselves must be used to provide a link for true zero level between all ports. The method used to obtain this common datum, viz. the mean undisturbed sea level over the period, is as follows. For each port the mean residual for the period 11 to 30 December was calculated and the results are plotted in figure 1 as ordinates, with distances along the coastline as abscissae. The majority of these mean residuals are referred to arbitrary datums. For each pair of adjacent ports the mean residuals were also computed for relatively undisturbed periods (table 1, column (*e*)), the periods chosen sometimes differing from pair to pair. The corresponding differences in mean residual are entered in column (*f*). The process was carried across the sea between Dover and Ostend. From the method of reduction employed it was known that the

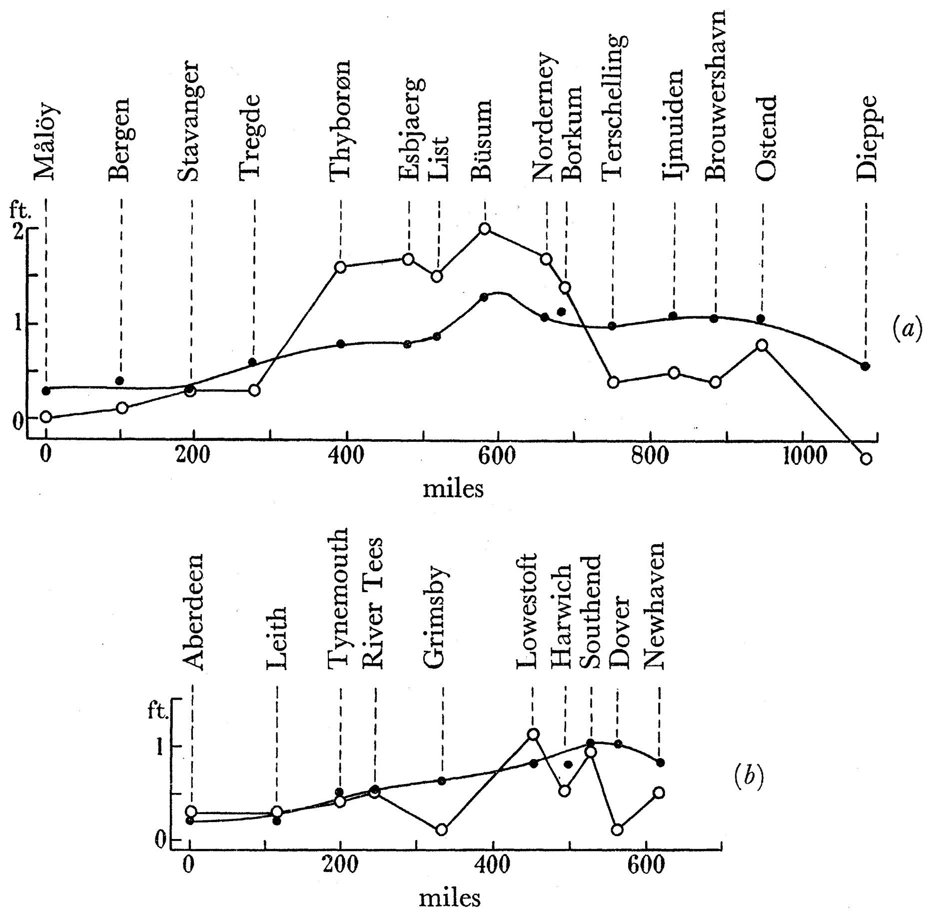

FIGURE 1. Mean residuals for 11 to 30 December 1954. ○, to arbitrary datums; ●, to common datum.

residuals for Aberdeen, Tynemouth, River Tees and Southend were reliably referred to the best probable estimate of the undisturbed mean level, and hence corrections to the residuals for all places were obtained by making the corrections for these four ports as small as possible. The corrections are given in column (*g*), and when applied to the uncorrected levels of figure 1 give a reasonably smooth curve for both shores of the North Sea and English Channel. The graphs show how the mean disturbance for the period increases towards the Straits of Dover, then decreases. The large local wind effect in the German Bight is clearly indicated.

The corrected residuals were then graphically smoothed to eliminate minor oscillations with periods of the order of 12 h or less, and the smoothed values are plotted at 3 h intervals, for selected times, in figures 6 and 7. With the exception of Grimsby, Harwich, Southend and the stations in the German Bight, the smoothing operation introduces negligible errors of judgement. For the stations mentioned, however, transient oscillations with periods of approximately 12 h are prominent during disturbed periods. These represent interaction between tide and surge, and are indicated in the appropriate diagrams by pecked lines.

In addition to observed heights of tide the National Institute of Oceanography kindly made available records of the difference in electric potential between the water on opposite sides of the English Channel, as recorded on the telephone cable running from St Margaret's Bay to Sangatte. Bowden (1956) has discussed their value as measurements of the mean flow of water through the Straits of Dover. The method of reduction referred to above is equally applicable to the cable measurements, and the residuals obtained are plotted along with the height residuals.

The height residuals have also been represented in the form of co-disturbance charts, figures 4 and 8. In drawing these charts local effects have been minimized as much as possible. For example, the residuals at Leith are not truly representative of the disturbance at the mouth of the Forth as typified by the average of the disturbances at Aberdeen and Tynemouth. Similarly, data for the island stations in the German Bight have been preferred to those for the coastal stations.

By integrating the residuals over the sea as indicated by the co-disturbance lines estimates have been obtained of the mean level of the sea, and these are given in relevant places in the text. The area over which the integration was effected was bounded by the lines Aberdeen/Stavanger in the north and Great Yarmouth/Terschelling in the south, excluding the Skagerrak and such shallow bodies of water as the Forth, the Wash and the shallowest part of the German Bight.

3. The effect of westerly winds

Previous statistical research [R] has indicated that there can be a differential effect between westerly winds over the sea proper and those over the area to the north of the British Isles. When obtaining surge forecasting equations for East Coast ports it was found that the surge generating potential of west winds inside the sea was rather indeterminate. A westerly wind tended to lower levels slightly on the East Coast if it was associated with a southerly wind component, and to raise them if associated with a northerly component. Many occasions were noted when strong winds over the sea from due west apparently caused a rise in level on the East Coast when a water gradient, positive to the east, would have been expected. The high degree of correlation between winds in the different areas concerned made it extremely difficult to confirm a differential effect.

Commencing early on 18 December, a strong westerly airstream covered the major part of the North Sea and the area to the north of the British Isles, varying somewhat in intensity and distribution until it veered on the morning of 20 December. Table 2 gives the mean residuals at groups of stations during the period, and the corresponding water gradients across the sea at three different latitudes. The main points of interest are:

(*a*) The level in the sea as a whole is raised by approximately 1 ft. This can only be attributed to the west winds in the north setting up a wind drift into the sea.

(*b*) In general there is a water gradient, positive to the east, which decreases southwards. This may reasonably be attributed to the winds over the sea which are weaker to the south.

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(*c*) There is a reversal in the direction of the water gradient at 00.00 and 06.00 h on 19 December, accompanied by a rise in level on the East Coast. Examination of the weather charts for the period shows that these occurrences may be correlated with an increase in wind strength to the north, which, causing an access in the southerly wind drift, generates a minor south-going surge. The surge is concentrated onto the East Coast by the earth's rotation.

Table 2. Mean residuals, 18 to 20 December 1954

date	hour	A	B	$B-A$	C	D	$D-C$	E	F	$F-E$
18 Dec.	0	0·1	1·2	1·1	0·0	1·2	1·2	1·1	0·9	−0·2
	6	0·6	1·4	0·8	0·7	1·0	0·3	0·0	−0·1	−0·1
	12	0·2	1·4	1·2	0·8	1·0	0·2	0·2	0·1	−0·1
	18	0·8	1·0	0·2	0·8	1·3	0·5	0·6	0·7	0·1
19	0	1·3	0·9	−0·4	1·4	1·6	0·2	0·9	1·3	0·4
	6	1·5	1·0	−0·5	1·6	1·5	−0·1	1·5	1·5	0·0
	12	0·8	1·2	0·4	1·0	1·4	0·4	1·5	1·4	−0·1
	18	0·7	1·2	0·5	0·8	1·4	0·6	0·9	1·0	0·1
20	0	0·6	1·0	0·4	0·9	1·0	0·1	1·1	0·7	−0·4
	6	0·8	1·0	0·2	1·0	0·9	−0·1	1·0	0·8	−0·2

A, Aberdeen and Leith; *B*, Tregde and Thyborøn; *C*, R. Tees and Tynemouth; *D*, Esbjaerg and List; *E*, Lowestoft, Harwich and Southend; *F*, Ijmuiden, Brouwershavn and Ostend. Units are feet.

It may be concluded that westerly wind systems between latitudes 58° and 65° N will raise the mean level of the North Sea, and the associated winds acting locally over the sea will set up a water gradient positive to the east, which, on the East Coast, may or may not cancel out the general rise. These conclusions will find confirmation in § 7.

4. 'Return' surges

In the course of preliminary statistical investigations into storm surges at Aberdeen and Immingham [R], correlation tables were compiled for all notable surges between 1930 and 1938 inclusive, the variables being the components of the tractive force of the wind at selected points in the North Sea (see figure 2) and the corresponding surge residuals. The correlations were performed for various time lags of residual on tractive force. Figure 3 illustrates the results. In all cases the ordinates are values of Σ(residual) $\times$ (tractive force component), i.e. rough approximations to the corresponding primary correlation coefficients. The residuals were not corrected for barometric pressure effects.

One of the most interesting features of the diagrams is a pronounced rise in level, following a lowering, for southerly winds. At Immingham the peaks follow the troughs by approximately 18 h, whilst at Aberdeen the trend is not so marked, presumably owing to the pressure effect there, which is large relative to the wind effect. The physical explanation offered at the time the investigation was carried out was that southerly winds drive water out of the North Sea, the water returning later down the East Coast in a manner closely analogous to that of an external surge. It is tempting to associate the value of 18 h with half the free period of the sea in a longitudinal direction, but in the framework of the proposed explanation this time lag probably corresponds to the average time taken by depressions to pass across the northern entrance to the sea.

Corkan (1950) first suggested a return flow to account for an otherwise unexplained small ise in level near Aberdeen on 8 January 1949, but this rise was soon swamped by a much arger surge generated by northerly winds. In subsequent research [R] many cases have

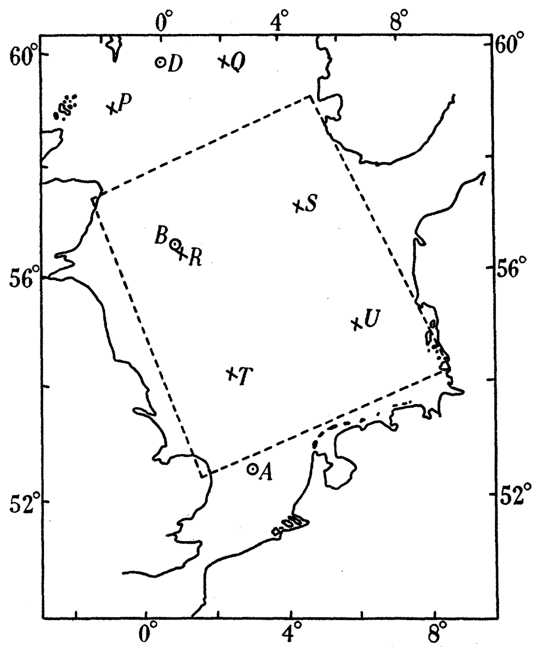

FIGURE 2. Points A, B, D, P, Q, R, S, T and U, in North Sea area, and schematic representation of the sea. The projection is equal area.

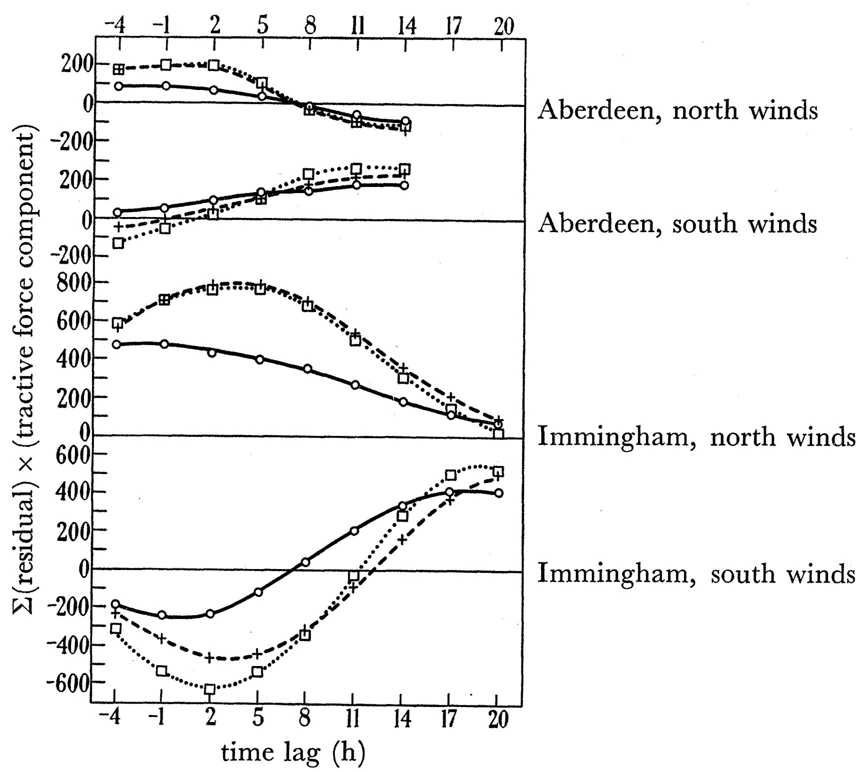

FIGURE 3. Correlations between residuals and components of wind stress for Immingham and Aberdeen, 1930 to 1938: —○—, point A; – – + – –, point B; ···□···, point D.

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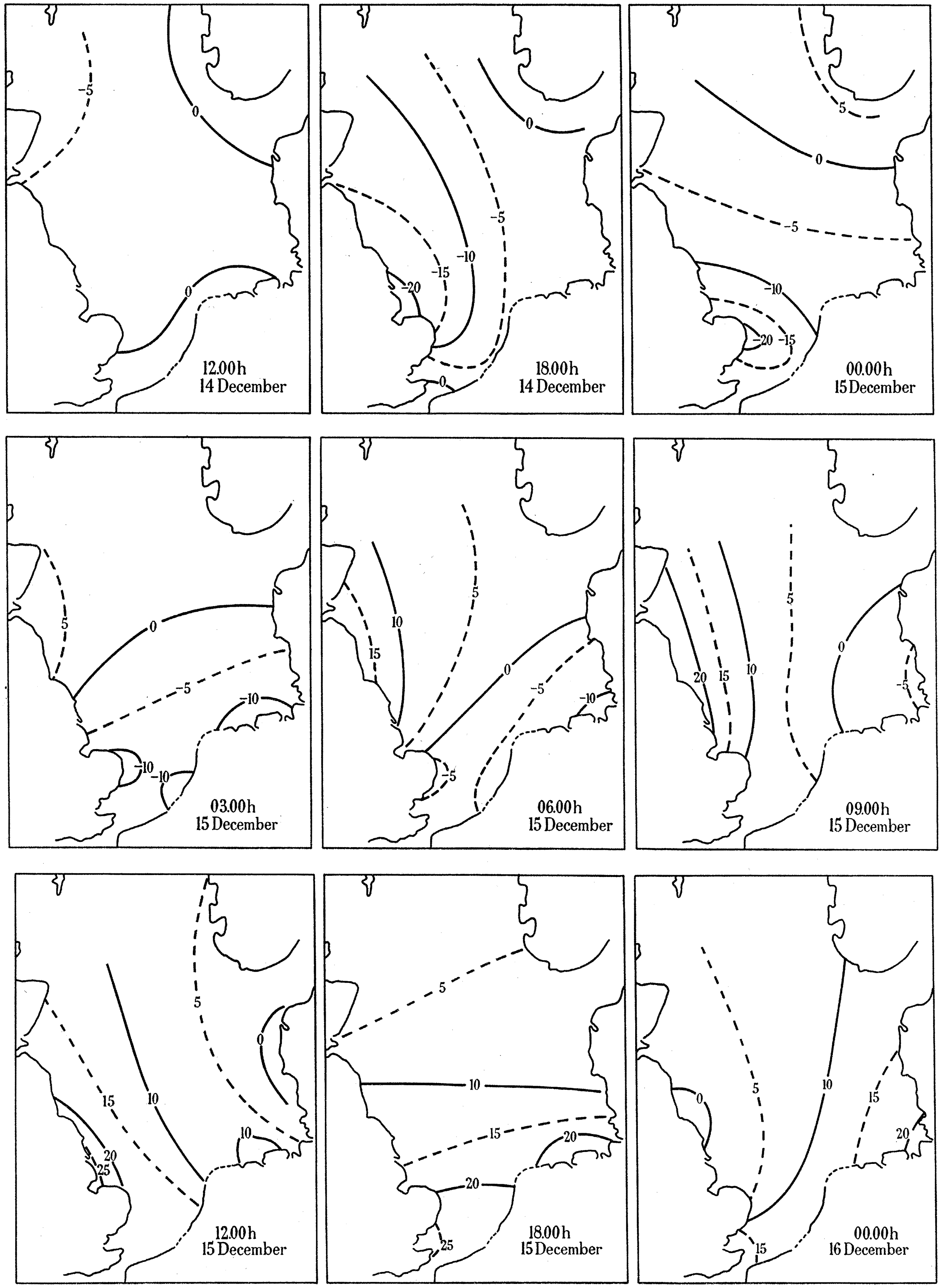

FIGURE 4. Co-disturbance lines, 12.00 h 14 December to 00.00 h 16 December 1954. (Unit $\frac{1}{10}$ ft.)

come to light in which positive surges have progressed down the East Coast following southerly winds which have subsequently died away completely.

The surge of 15 December 1954 offers evidence of a regional, qualitative nature to support the return surge theory. It cannot be considered wholly conclusive, however, for a reason which will emerge below.

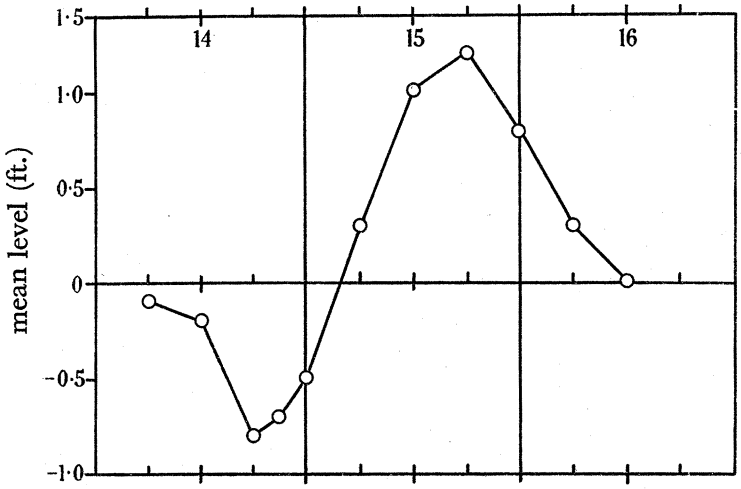

FIGURE 5. Mean level of the North Sea, 14 to 16 December 1954.

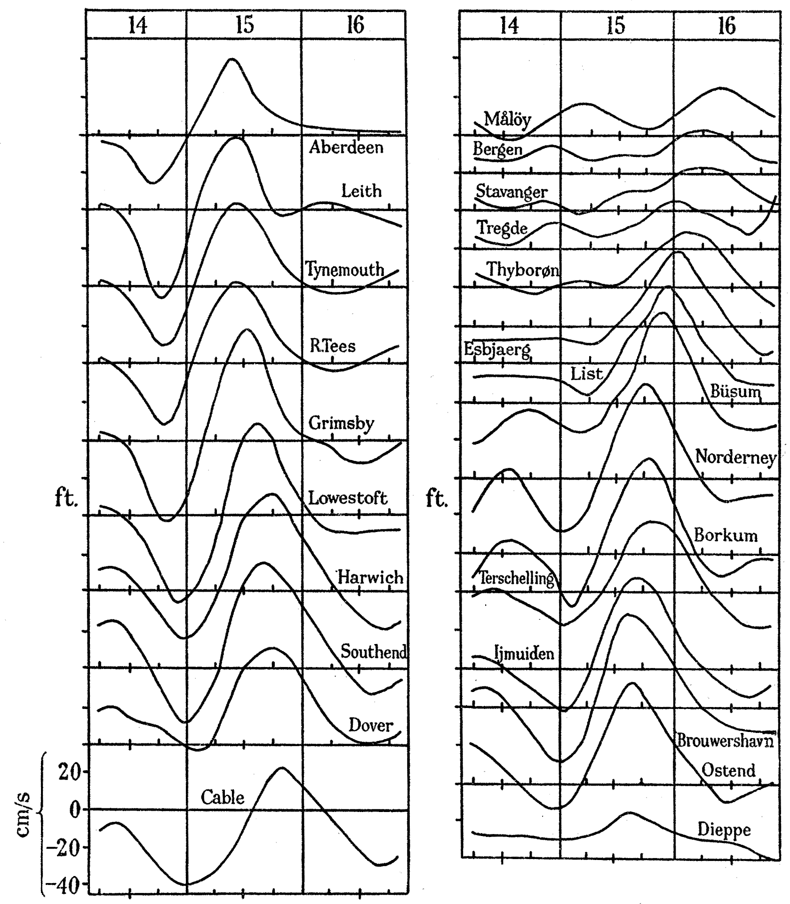

FIGURE 6. Smoothed residuals, 14 to 16 December 1954.

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14 *December*. A depression of some 980 mb travelling approximately east along latitude 60° N has by 12.00 h brought its associated south to south-west winds to bear upon the western North Sea, and as their effect is extended eastwards the resulting variations in level are as illustrated in the co-disturbance lines of figure 4. The variations in the mean level of the sea are shown in figure 5, and it will be seen that by 21.00 h the mean level is nearly 1 ft. below normal. The water gradient runs north-north-east and south-south-west and has reached a maximum. Shortly afterwards the flow through the Straits of Dover from the English Channel has also reached its maximum value. By midnight the depression begins to swing north-westwards, and at this time there is a slight tendency for the westerly winds to the north to vary in such a manner as to give rise to an external surge. From past experience the author would judge that such a surge would be small, considerably less than the surge observed (figure 6). In private correspondence, Crease writes that his work (1956) on the generation of external surges by travelling depressions would lead him to expect a small surge to reach Aberdeen at about 02.00 or 03.00 h on 15 December, whereas the peak disturbance recorded there is some 6 h later.

15 *and* 16 *December*. Ignoring this improbable, alternative explanation, the subsequent co-disturbance lines show how the water gradient decays and the water is returned to the East Coast as a positive surge which progresses anticlockwise around the North Sea. After 06.00 h the winds over the sea are too light to be capable of generating this surge, though some of the rise in level must be attributed to the westerly winds to the north. The progressive nature of the surge is clearly revealed in figure 6, and its partial transmission into the English Channel is shown by the residuals for Newhaven and by the cable records.

5. Qualitative description of disturbances, 20 to 25 December

For this period the residuals are plotted in figure 7, and the co-disturbance lines in figure 8. The mean level of the sea will be found in figure 9.

20 *December*. The westerly winds hitherto prevailing over the region have caused a general rise in level in the North Sea, with levels higher on the continental coastline (12.00 h). As the winds further north veer and strengthen (12.00 to 18.00 h) a surge is produced in the North Sea with a maximum water gradient at midnight lying approximately along the longitudinal axis of the sea.

21 *December*. This gradient now begins to decay as the winds back to the west. As the water flows out of the sea in a northerly direction the geostrophic force concentrates this flow onto the continental coast, with the result that the co-disturbance lines rotate in an anticlockwise direction, so that by 12.00 h they run north and south. This easterly gradient must also contain a contribution from the local westerly winds which are of the order of Beaufort force 6 to 7. By this time the depression of 960 to 970 mb which has been travelling south-east from Iceland is situated just north of the entrance to the sea, and the effect of the gale force north-westerly winds in its rear is clearly seen in the co-disturbance chart for 18.00 h as an external surge progressing down the East Coast. These meteorological conditions are in conformity with Corkan's observations on, and Crease's travelling wind theory of, external surges. In the southern part of the sea the easterly water gradient mentioned earlier is still present, but by 21.00 h is virtually obliterated.

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22 *December*. As the north-westerly winds spread over the whole sea, the external surge becomes indistinguishable from the main disturbance which grows until a maximum gradient is attained by 06.00 h. At this time the co-disturbance lines are virtually orthogonal

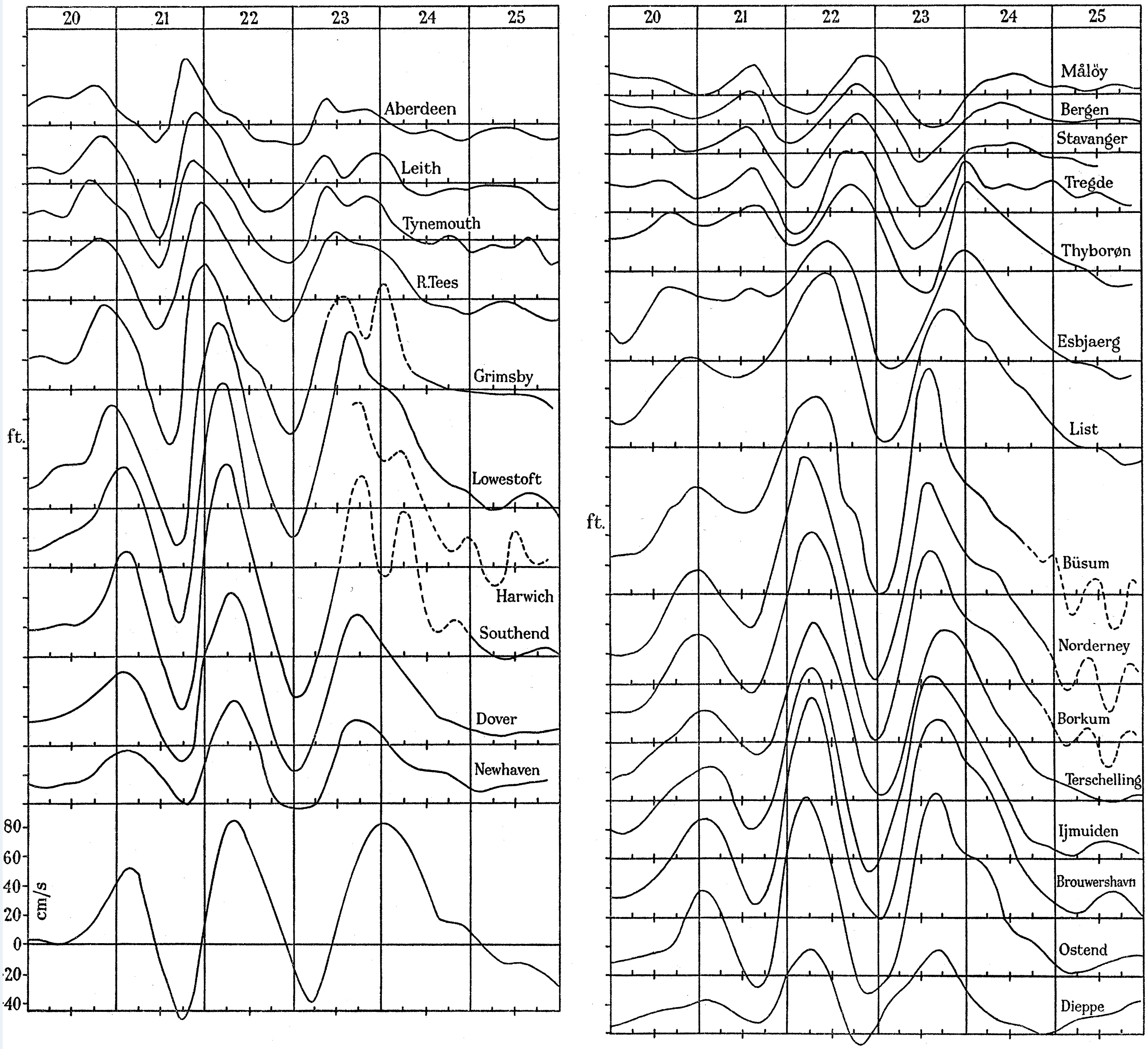

FIGURE 7. Smoothed residuals, 20 to 25 December 1954.

with the isobars, so that a quasi-stationary condition has been reached. The mean level of the sea is 2·5 ft. above normal, and the outflow through the Straits of Dover has almost reached its peak. The greatest levels are now being experienced on the German, Netherlands and South-East coasts. As the depression travels eastwards over southern Sweden the winds over the sea and its approaches weaken and back, and the surge begins to decay. The

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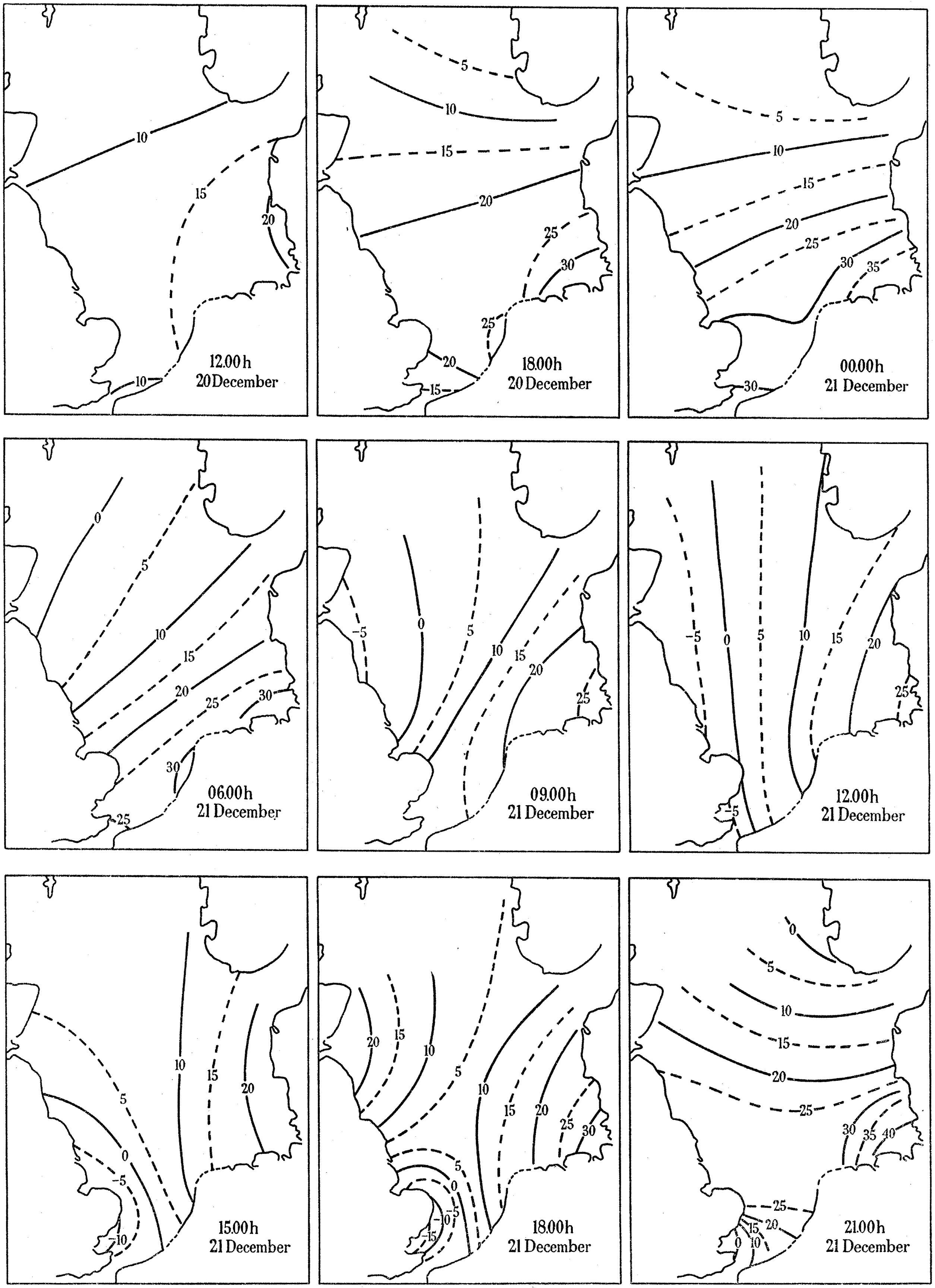

FIGURE 8. (For legend see p. 152.)

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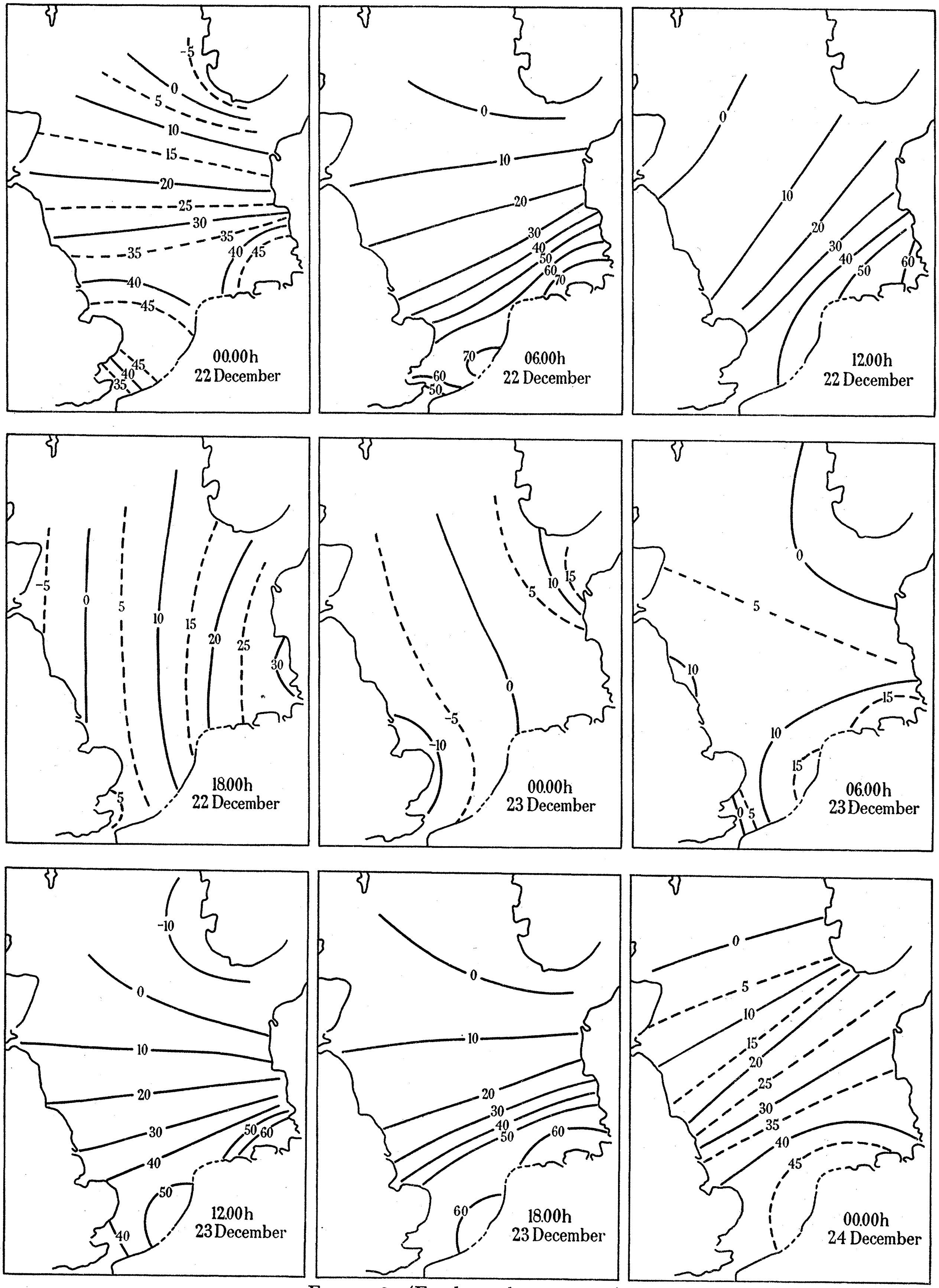

FIGURE 8. (For legend see p. 152.)

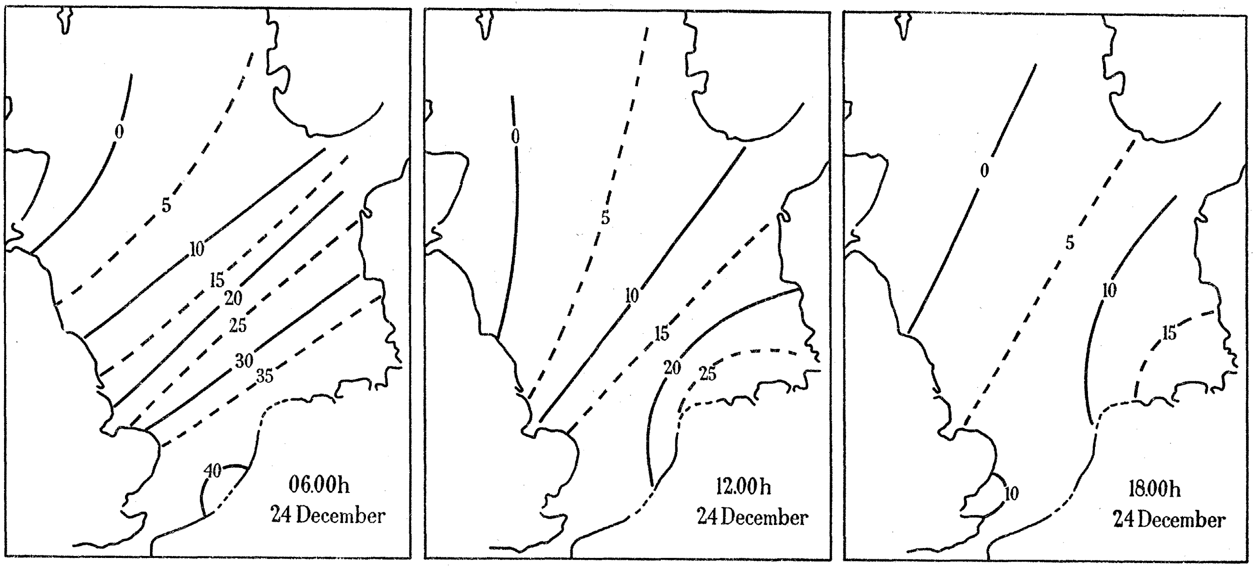

FIGURE 8. Co-disturbance lines, 12.00 h 20 December to 18.00 h 24 December 1954. (Unit $\frac{1}{10}$ ft.)

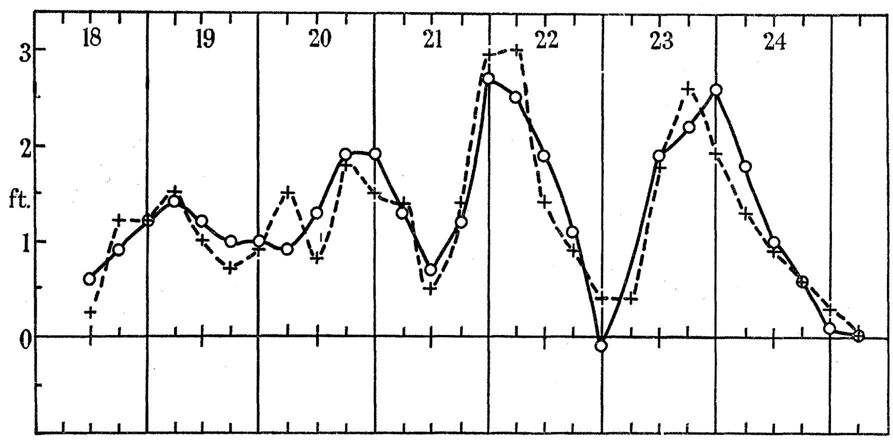

FIGURE 9. Mean level of the North Sea, 18 to 25 December 1954.
—o—, observed; – –+– –, computed.

co-disturbance diagrams show the similarity between this decay and that of the preceding surge. Once again the water flows northwards and, under the influence of the earth's rotation, the flow is concentrated to the east and a transverse gradient is set up, with a resulting anticlockwise rotation of the co-disturbance lines. This process continues throughout the remainder of the day. The highest level can be traced as it travels north from the German Bight. By midnight there only remains a water gradient across the sea which may be attributed to the westerly winds over the sea, and the residue of the surge near the Skagerrak. A point of interest here is that the local west winds have caused the levels on the East Coast to fall below zero, there being no strong westerly winds to the north of the sea to counteract this effect by generating a southerly wind drift.

23 *December*. During the night of 22 December a complex low-pressure system forms in the approaches to the North Sea, and as it passes eastwards gale force north-westerly winds rapidly spread over the sea. By 06.00 h they are raising the levels on the coasts of Holland and Germany. Unlike the previous surge, however, there is no indication of a disturbance

travelling down the East Coast. The meteorological conditions have not been favourable for either an external or a return surge; nevertheless, the charts show that during the growth of this surge the geostrophic effect is prominent in developing higher levels to the right of the wind in the main body of the sea. By 18.00 h the maximum water gradient exists in an approximately north and south direction, but by this time the winds are beginning to weaken. The mean level over the sea reaches a maximum of 2·5 ft. near midnight, and a tendency for the co-disturbance lines to rotate anticlockwise indicates that already some excess water is trending north along the continental coast.

24 *and* 25 *December*. Throughout the next 2 days the decay of the surge may be traced, and it will be noted that because the winds over the sea do not back as quickly as with the previous surge, the rotation of the co-disturbance lines is not as marked.

6. Water gradients in the North Sea, 20 to 25 December

6·1. *Longitudinal water gradient*

To simplify the problem we may consider the sea to be replaced by a rectangle with its length at an angle of 23° to the meridian (figure 2). The mean residual along each boundary of the rectangle was extracted from the co-disturbance charts, using only the indications of the co-disturbance lines. With axes Ox, Oy along the sides of the rectangle and the origin at the north-west corner, the mean longitudinal and transverse gradients are denoted by $\Delta\zeta_y$ and $\Delta\zeta_x$, respectively. The subscript t will be used to denote time in hours.

Estimates of the geostrophic wind were extracted from synoptic charts, kindly supplied by the Meteorological Office, Speke Airport, by reading off the northerly and easterly pressure gradients over distances of 250 miles at points P, Q, R, S, T and U (figure 2). As the curvature of the isobars was not appreciable during the period under discussion no attempt has been made to compute the gradient wind. Assuming the tractive force of the wind on the sea surface to be proportional to the square of the wind velocity, southerly (S) and easterly (E) components of the tractive force were calculated, in arbitrary units. For a wind from the south-east S and E would both be negative.

Let

$$\Delta\zeta_{y,t} = aS_{t-\alpha} + bE_{t-\alpha}, \tag{1}$$

where α is a time lag and a, b are constants. With the data for 20 to 25 December, and using S, E averaged for points P to U, the correlation coefficients between opposite sides of equation (1) for different values of α are given in table 3. For $\alpha = 6$ h then $a = 0{\cdot}196 \pm 0{\cdot}011$, $b = 0{\cdot}016 \pm 0{\cdot}016$. The most effective direction of the geostrophic wind is thus from N 5° W, and if we assume the direction of the maximum water gradient to coincide with that of the surface wind we deduce an inclination of 18° between geostrophic and surface winds during the period, a value which is in good agreement with that generally accepted. Observed values of $\Delta\zeta_y$ are given in table 4 and figure 10, together with values computed from equations (1) with $\alpha = 6$ h.

The arbitrary units used for S and E may be transferred into the c.g.s. system through the geostrophic wind equation

$$V = G/2\omega \sin\phi,$$

so that

$$V^2 = 2{\cdot}85 \times 10^5 (S^2 + E^2)^{\frac{1}{2}},$$

where V is the geostrophic wind velocity in cm/s, G is the pressure gradient in mb/250 miles, ω is the angular speed of rotation of the earth, and ϕ is the latitude. Equation (1) may be written

$$\Delta\zeta_y = 2{\cdot}11 V^2 \cos(\theta - 5^\circ) \times 10^{-5},$$

where $\Delta\zeta_y$ and V are in c.g.s. units and $\theta = \tan^{-1} E/S$. Assuming the surface wind velocity V_s to be two-thirds that of the geostrophic wind,

$$\Delta\zeta_y = 4{\cdot}75 V_s^2 \cos(\theta - 5^\circ) \times 10^{-5}. \tag{2}$$

TABLE 3. CORRELATION COEFFICIENTS FROM EQUATION (1)

α	3	6	9
r	0·92	0·96	0·87

TABLE 4. OBSERVED (o) AND COMPUTED (c) VALUES OF $\Delta\zeta_y$ AND $\Delta\zeta_x$

		$\Delta\zeta_y$		$\Delta\zeta_x$	
date	hour	(o)	(c)	(o)	(c)
20 Dec.	0	0·1	0·2	0·1	—
	6	0·2	0·4	0·0	—
	12	0·5	0·4	0·4	0·5
	18	1·4	1·5	−0·1	0·0
21	0	2·5	2·1	0·0	−0·3
	6	2·3	1·9	0·8	0·6
	12	0·5	0·2	2·2	1·9
	18	0·4	0·6	0·7	1·1
22	0	3·7	4·0	−1·2	−0·9
	6	5·9	5·7	0·1	−0·3
	12	3·6	4·0	2·1	1·9
	18	1·1	1·7	2·4	2·3
23	0	−0·2	0·6	1·5	1·2
	6	1·1	1·0	−0·7	−0·5
	12	4·9	3·2	−1·0	−1·6
	18	5·2	5·2	−0·3	0·3
24	0	4·0	4·0	0·7	1·3
	6	2·9	3·1	1·2	1·0
	12	2·0	2·5	1·2	0·8
	18	1·3	1·9	0·9	0·7
25	0	0·7	1·1	0·2	0·6
	6	0·3	0·6	0·3	0·4
	12	0·4	0·6	0·4	0·0
	18	0·4	0·2	0·0	—

Units are feet.

For stationary conditions, the slope of the water surface down the axis of the sea may be represented by

$$\frac{\Delta\zeta}{\Delta y} = \frac{n\gamma^2}{gh}\frac{\rho_a}{\rho} V_s^2 \cos(\theta - 5^\circ), \tag{3}$$

where h = the mean depth of water,

ρ_a/ρ = the ratio of densities of air and water,

γ^2 = frictional 'constant' defined by the equation $T = \gamma^2 \rho_a V_s^2$ (T is the tractive force of the wind), and

n is a constant which is theoretically 1·5 for no bottom current and unity when there is no bottom friction.

Taking $\Delta y = 6{\cdot}1 \times 10^7$ cm, $h = 6{\cdot}5 \times 10^3$ cm, $\rho_a/\rho = 1{\cdot}2 \times 10^{-3}$ and $n = 1{\cdot}5$,

then $$\gamma^2 = 2{\cdot}7 \times 10^{-3}.$$

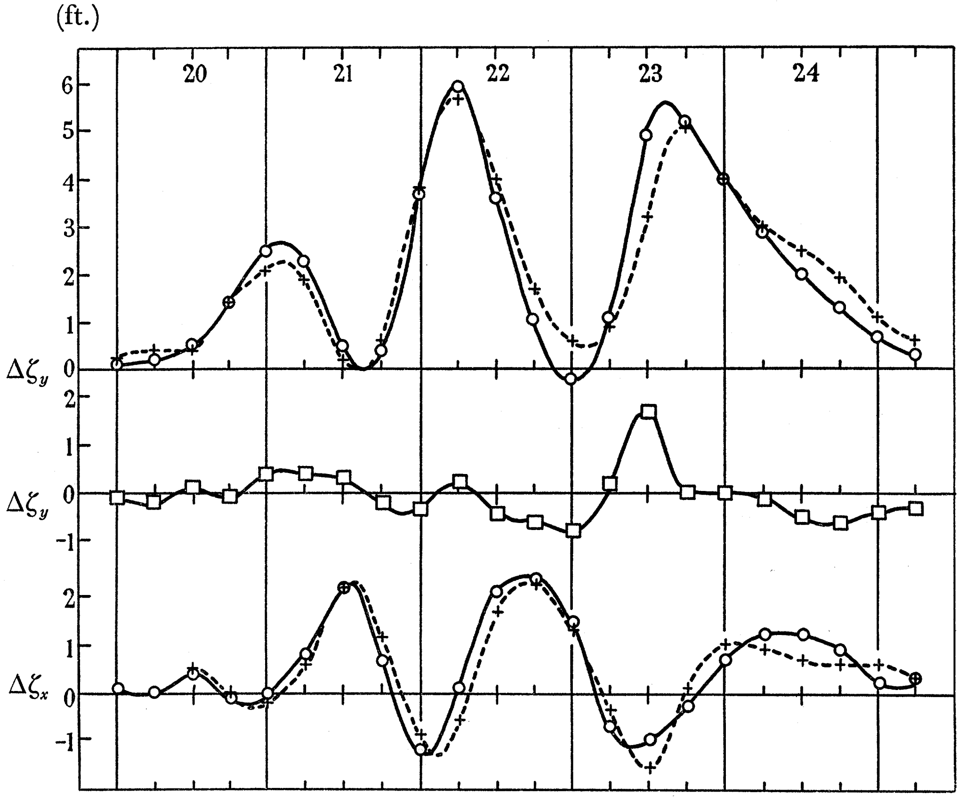

FIGURE 10. Water gradients, 20 to 25 December 1954. $\Delta\zeta_y$ = longitudinal gradient, $\Delta\zeta_x$ = transverse gradient. —○—, observed; – – + – –, computed; —□—, observed – computed.

TABLE 5. CORRELATION COEFFICIENTS FROM EQUATION (4)

β \ α	0	3	6
0	0·84	0·79	0·76
3	0·93	0·94	0·92
6	0·67	0·68	—

6·2. *Transverse water gradient*

In §§ 3 and 4 it was demonstrated that transverse gradients could be produced both by westerly winds inside the sea and by geostrophic forces. This may be expressed in the form

$$\Delta\zeta_{x,\,t} = aX_{t-\alpha} + b\,\mathrm{d}(\Delta\zeta_{y,\,t-\beta})/\mathrm{d}t. \tag{4}$$

It will be assumed that the angle between the geostrophic wind and the maximum water gradient is 18°, as found in § 6·1, and X is the component, parallel to Ox, of the square of the geostrophic wind velocity. X has been computed from values of S and E averaged for points R to U, and is in the same units as S and E. $\mathrm{d}(\Delta\zeta_y)/\mathrm{d}t$ was computed in units of ft./6 h from observed values of $\Delta\zeta_y$ by the usual central difference formula. $\Delta\zeta_x$ is in feet. For various values of the time lags α and β the correlation coefficients between opposite sides of the above equation are given in table 5. For $\alpha = \beta = 3$ h, $a = 0{\cdot}030 \pm 0{\cdot}014$, $b = -0{\cdot}059 \pm 0{\cdot}005$.

Observed values of $\Delta\zeta_x$ are given in table 4 and figure 10, together with values computed from equation (4) with $\alpha = \beta = 3$ h. The computations reveal that for the surges examined the maximum contribution to $\Delta\zeta_x$ from the geostrophic effect exceeds 2 ft., more than double the maximum direct effect of the westerly winds during the period.

A similar calculation to that of § 6·1, with $\Delta x = 4{\cdot}7 \times 10^3$ cm gives a value for γ^2 of $0{\cdot}8 \times 10^{-3}$. The relatively large standard error of the regression coefficient a indicates that this estimate for γ^2 is unreliable. This may be due to the assumption that transverse water gradients do not deviate appreciably in direction from the surface winds producing them.

7. Mean level of the North Sea, 18 to 25 December

It was shown in § 3 that the mean level ($\bar{\zeta}$) of the sea is primarily determined by the south-going wind drift from the north and north-west. We may write

$$\bar{\zeta}_t = aS'_{t-\alpha} + bE'_{t-\alpha} + cY_{t-\beta}. \tag{5}$$

The first two terms on the right-hand side represent the wind drift due to winds at the point 62° N, 10° W, i.e. midway between Iceland and Scotland, and the last term represents the drift due to winds in the northern part of the sea. Y is the component down the axis of the sea of the square of the geostrophic wind velocity, as derived from the mean of the pressure gradients at points P to S. A graphical comparison of the data for 21 to 25 December

Table 6. Correlation coefficients from equation (5)

α	6	12	18
r	0·86	0·91	0·86

reveals that the last term is then predominant, and that a good estimate for the time-lag β is 6 h. The correlation coefficients from this data for 18 to 25 December for various values of the lag α are given in table 6. For $\alpha = 12$ h, $\beta = 6$ h then

$$a = 0{\cdot}016 \pm 0{\cdot}011, \quad b = 0{\cdot}031 \pm 0{\cdot}007, \quad c = 0{\cdot}053 \pm 0{\cdot}013.$$

These results indicate that winds to the north-west of the British Isles play a not inconsiderable role in raising the mean level of the sea, and the most effective direction for the geostrophic wind in this area is from W 27° N. Observed values of $\bar{\zeta}$ are plotted in figure 9, together with values computed from equation (5) with $\alpha = 12$ h, $\beta = 6$ h.

8. Flow of water through the Straits of Dover, 12 to 28 December

It has been shown (Rossiter 1954) that the existence of the Straits of Dover is of practical importance in providing an outlet for some proportion of a positive surge in the southern North Sea. One consequence of this is that such surges can be propagated from the North Sea into the English Channel, and it has been found [R] that the majority of disturbances occurring at Newhaven have travelled from Lowestoft in 4 h and have been reduced in magnitude by 50 %. Using the indications of the measurements taken by the telephone cable across the Straits, Bowden (1956) has represented the mean cross-sectional flow (U cm/s) by the linear equation

$$U = aV^2 \cos\psi + b\Delta\zeta, \tag{6}$$

where V m/s is the wind speed at Lympne, and ψ the angle between the direction of the wind vector and the normal to the cable. $\Delta\zeta$ is the difference between the mean residuals (cm) along the sections Shoreham/Dieppe and Lowestoft/Flushing. Values of the regression coefficients a and b were obtained for 25 h mean values of the data, for differing periods, and are reproduced in table 7. The total correlation coefficients are also shown. The data for 12 to 29 December 1954 have also been treated in a similar fashion, with the differences that Shoeburyness wind data have been used in the absence of those for Lympne, and Shoreham has been replaced by Newhaven. The results are shown in table 7. The higher value of the total correlation coefficient (0·96) may be attributed to the shorter span of records used; the value of a is appreciably smaller than that obtained by Bowden, and this will be discussed below; the value of b is in reasonable agreement with his figures.

TABLE 7. VALUES OF COEFFICIENTS IN EQUATION (6), AND ESTIMATES OF γ^2 AND k

period	a	b	r	γ^2	k
15 Sept. 1953 to 31 Mar. 1954 (Bowden 1956)	$0{\cdot}37 \pm 0{\cdot}03$	$0{\cdot}70 \pm 0{\cdot}07$	0·88	$4{\cdot}5 \times 10^{-3}$	$3{\cdot}8 \times 10^{-3}$
12 to 28 Dec. 1954 (25-hourly means)	$0{\cdot}19 \pm 0{\cdot}05$	$0{\cdot}76 \pm 0{\cdot}05$	0·96	$2{\cdot}1 \times 10^{-3}$	$3{\cdot}5 \times 10^{-3}$
12 to 28 Dec. 1954 (3-hourly means)	$0{\cdot}16 \pm 0{\cdot}02$	$0{\cdot}59 \pm 0{\cdot}02$	0·95	$2{\cdot}3 \times 10^{-3}$	$4{\cdot}5 \times 10^{-3}$

It is of interest to determine whether the relationship (6) is also valid if the variables are taken at shorter intervals of time than a day. The correlations were accordingly carried out with U, $V^2 \cos\psi$ and $\Delta\zeta$, at intervals of 3 h, for the period 12 to 29 December. The total correlation coefficients indicated that the best time lags to be employed were 0 to 3 h for $V^2 \cos\psi$, and 3 h for $\Delta\zeta$. The values for a, b and r, corresponding to lags of zero and 3 h for $V^2 \cos\psi$ and $\Delta\zeta$, respectively, are given in table 7. An important by-product of these correlations was confirmation of the datum used in the reduction of the cable measurements.

Equating the coefficients a and b with quantities derived from a theoretical treatment of the dynamics of flow in a channel of variable cross-section, in the presence of a tidal current, Bowden then proceeded to obtain estimates for the wind stress coefficient γ^2 and the coefficient of bottom friction k. These, together with estimates obtained in the same manner from the data of the present paper, are given in table 7. Values of γ^2 obtained by the writer are much closer to the magnitude generally expected than those of Bowden, and it may be that the exposure of the Shoeburyness anemometer makes its records more representative of the winds over the Straits than are the Lympne records. Values of k agree with Bowden's results in being approximately twice as large as would be expected, and it is noticeable that k is greater for data taken at intervals of 3 h than for data at intervals of a day. This is in accord with his suggestion that even for the latter data a sufficiently close approximation to steady-state conditions is not realized.

9. OSCILLATORY OR SEICHE MOTION IN THE NORTH SEA

It has long been known that forced motion of bodies of water can be accompanied by seiche motion. Seiches on lakes have been observed and investigated from the 15th century onwards, and more recently by Forel, Chrystal and many others. Merian (1828) obtained a formula for calculating the period of seiches in a uniform, rectangular basin, which has formed a basis for investigations by Nomitsu (1935), Stenij (1936), Palmèn & Laurila

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(1938) and others regarding storm surge oscillations in the Baltic and in lakes. Thorade (1923) observed that residuals at German North Sea ports were of an oscillatory nature similar to those at Blyth, Grimsby, Hirtshals and Frederikshaven, and came to the conclusion that wind effect is not aperiodic but develops in the form of oscillations. Proudman & Doodson (1924) obtained theoretical solutions for the damped oscillatory motion caused by wind—but equally applicable if the generating force is atmospheric pressure—for a closed rectangular basin on a non-rotating earth. They considered the mode in which a steady state is reached and gave a formula for the general variable state. Nomitsu (1934, 1935), taking into account geostrophic effects, confirmed their work. The only published practical applications of the oscillatory theory appear to be those of Nomitsu (1935) for Lake Biwa, Schalkwijk (1947) for the Netherlands coast, and Kivisild (1954) for Lake Okeechobee. In unpublished work on surges at Southend, Doodson & Corkan used the theory with great success in many instances, but Corkan ultimately considered the damping factor to be such as to allow neglect of all oscillations after the first peak of a positive surge. Seiche motion is essentially a direct result of water possessing inertia, and the time lag between the generating wind and the resulting water gradient is a prominent manifestation of this property. Formulae produced in the theoretical work referred to above clearly bring out this feature, and it may be shown that the time lag is approximately one-quarter of the free period of the body of water. The time lags of 6 h for the longitudinal gradient and 3 h for the transverse gradient obtained in § 6 are in reasonable agreement with this statement, as the free periods are respectively of the order of 30 to 40 h and 12 h. The question at issue, then, is not whether seiching does take place but the amount of damping associated with it.

In an attempt to confirm or deny the validity of Corkan's neglect of seiching a new investigation was undertaken by the present author. An exhaustive search was made for damped oscillations in many years of residuals for Aberdeen, Lowestoft and Southend. The main conclusion reached was that for northerly winds any oscillations which may exist are so heavily damped as to be indiscernible among other second-order effects such as interaction between tide and surge. Analysis of residuals at all three ports revealed no lack of transient oscillations in the period band of 24 to 40 h, but no consistent results could be obtained. Darbyshire & Darbyshire (1956) arrived at a similar conclusion. Reference may also be made here to the data of figure 3. There is no indication in the diagram for Immingham of the rise produced by northerly winds being followed by a lowering of level.

In a study of the twin surges of 21 to 24 December 1954, however, Weenink (1956) finds evidence of an appreciable oscillatory contribution to the residuals for Hoek van Holland. The validity of his results rests to a considerable extent upon the accuracy of the equilibrium wind effect as calculated from meteorological observations using Schalkwijk's method. In the derivation of his formula, Schalkwijk ignored geostrophic effects in the North Sea, and the present investigation does not contradict this, for the inclination between the water gradient and the surface wind when quasi-stationary conditions exist is small. The geostrophic effect when gradients are growing or decaying has been shown to be considerable, however, and these cannot have been allowed for in Weenink's work (nor in Schalkwijk's own attempt to introduce oscillatory motion into surge forecasting), and the results of the present paper would suggest that this omission could largely be responsible for the deviations in the residuals which Weenink attributes to oscillatory motion. It is true, that the geostrophic effect of the currents flowing along the South East and Netherlands coasts is very small, as the correlation coefficient between the cable measurements and the water gradients across the southern arm of the North Sea, after allowing for the local wind effect, is only 0·5. The levels in this area, however, must very largely be controlled by the phenomena occurring in the main body of the sea.

In view of the divergence of opinions about the magnitude of oscillations in the North Sea it therefore becomes of interest to examine whether the data presented here support either argument. If appreciable seiching existed it would be expected that as the water flowed north after each positive surge the mean level of the sea would fall below its equilibrium value, the defect being a maximum near the troughs of figure 9. It will be seen that this is not so. Seiching would also be expected to appear as a deviation between the observed and computed values of the longitudinal water gradient with a period of some 30 to 40 h. Figure 10 reveals a tendency for such an oscillation to occur, starting on 22 December. The magnitude of this oscillation, however, would be reduced if a time lag of 4 to 5 h (as indicated in § 6·1) were taken in the computations instead of the lag of 6 h used for ease of computation.

Much of the evidence put forward here appears to indicate that *positive* surges in the North Sea develop in an oscillatory manner which is so heavily damped that the disturbances produced may effectively be represented by assuming equilibrium conditions. The qualitative descriptions of the water movements given earlier, and the correlations between winds and water gradients and between winds and the mean level of the sea, are compatible with one another, and with the concept of gradients being generated and then decaying under the influence of wind and the earth's rotation. The geostrophic force produces considerable transverse gradients during the growth and decay of the surges, but has no marked effect upon quasi-stationary conditions. The general impression produced is one of solitary Kelvin waves. *Negative* surges, however, exhibit a strong tendency towards oscillations, as is evidenced in the discussion of § 4.

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